

Structural features that predict real-value fluctuations of globular proteins

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ABSTRACT

It is crucial to consider dynamics for understanding the biological function of proteins. We used a large number of molecular dynamics (MD) trajectories of nonhomologous proteins as references and examined static structural features of proteins that are most relevant to fluctuations. We examined correlation of individual structural features with fluctuations and further investigated effective combinations of features for predicting the real value of residue fluctuations using the support vector regression (SVR). It was found that some structural features have higher correlation than crystallographic B-factors with fluctuations observed in MD trajectories. Moreover, SVR that uses combinations of static structural features showed accurate prediction of fluctuations with an average Pearson's correlation coefficient of 0.669 and a root mean square error of 1.04 Å. This correlation coefficient is higher than the one observed in predictions by the Gaussian network model (GNM). An advantage of the developed method over the GNMs is that the former predicts the real value of fluctuation. The results help improve our understanding of relationships between protein structure and fluctuation. Furthermore, the developed method provides a convienient practial way to predict fluctuations of proteins using easily computed static structural features of proteins.

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Key words: protein flexibility; protein dynamics; structure-dynamics relationship; support vector regression; molecular dynamics; fluctuation prediction.

INTRODUCTION

Thanks to worldwide efforts in structural genomics, 1-3 we now know over 75,000 protein tertiary structures.⁴ This number is only a small fraction when compared with the number of known protein sequences. Computational methods can predict structures for more than a half of newly sequenced proteins by means of template-based modeling with a sufficiently high accuracy. 5-8 For some of the remaining proteins, it is possible to predict their structures in a de novo fashion if they are small and structurally simple. 9-14 Thus, the problem of protein structure prediction is practically gradually being solved, and it may be completely solved in the near future. Obviously, for the most difficult (and "atypical") cases of monomeric structures and to a much larger extent for the plethora of possible protein-protein (protein-nucleic acid, protein-carbohydrate, etc.) complexes, structure prediction will remain a challenging task for decades. 9,15-17 The knowledge of protein tertiary structures facilitates fast developments in various branches of molecular medicine and biotechnology. 18,19 It, however, becomes more and more obvious that to understand the underlying molecular mechanisms of life, we need to see biomolecules "in action."

Protein dynamics, resulting from a specific flexibility of their structures, has drawn much attention recently in both theoretical and experimental molecular biology. Studies of dynamics of protein structures and their assemblies are important for understanding the mechanisms of protein function in various cellular processes, 20,21 in particular, ligand binding, enzymatic reactions, 22 conformational diseases, 23 and protein–protein interaction. The understanding of protein flexibility is also important for practical applications such as development of computer-aided methods of enzyme design 25,26 and drug development.

In X-ray protein crystallography, which determines the Cartesian coordinates of atoms in proteins, uncertainties/fluctuations of atomic positions are provided in the form of *B*-factors. ²⁸ The *B*-factor measures the mobility of atoms, but it also reflects some inherent aspects of crystallographic

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techniques. Moreover, fluctuations estimated by B-factors are influenced by the molecular environment of the crystal structure. Protein mobility in solution could differ qualitatively from that in a crystal. Eastman et al.²⁹ showed that B-factors are an accurate measure of fluctuations for stable parts of proteins, but significantly underestimate motion in flexible regions. Somewhat more straightforward measures of structure fluctuations could be derived from nucleic magnetic resonance (NMR) experiments, although resulting estimates can be flawed by various limitations of actual measurements and by the computational schemes of their interpretation. 30-33 Therefore, these methods do not fully reflect actual fluctuations of proteins.

Molecular dynamics (MD) is the most straightforward method for theoretical studies of dynamic aspects of molecular systems. Because of the progress in computing technology, it is now practical to simulate protein systems in a timescale of tens of nanoseconds. Nevertheless, such simulations remain costly. With a significantly less computational requirement, the internal motion of a protein can be approximated by the normal mode analysis of a harmonic model of proteins.³⁴ Another possibility is to use simulations using coarsegrained representations of protein structures. A simple approach is the Gaussian Network Model (GNM) and its derivatives.^{35–38} Long-time simulation at an intermediate resolution can be achieved using simplified protein models such as UNRES³⁹ and CABS.⁴⁰ These models enable a low-resolution study of dynamics (or stochastic dynamics) in timescales by a few orders of magnitude longer than possible by all-atom MD.41-44 A weak point of studying dynamics using coarse-grained models is a lack of straightforward scaling between the models' time and the real time. Thus, all-atom MD simulations should always be used as a reference for coarse-grained dynamics.

A number of computational methods for predicting protein fluctuations have been published; however, almost all of them evaluated their prediction results mainly in comparison with the crystallographic B-factor of proteins. As discussed earlier, the B-factor does not fully capture the mobility of proteins in solution. As we show in this work, the fluctuations observed in MD and the B-factor correlate rather poorly, as was also concluded in a previous work.²⁹

There are a series of works that use GNM or its variants for predicting B-factors of proteins. 35,38,45,46 Micheletti et al.⁴⁷ extended GNM by adding Cβ atoms (βGM). The fluctuations of residues predicted by βGM were compared to the fluctuations from the MD simulation of HIV-1 protease. The self-consistent pair contact probability method, which is similar in its spirit to GNM, was used to predict fluctuations and compared with B-factors. 48 Zhou and coworkers 49 developed an all-atom mean-field model to predict fluctuations.

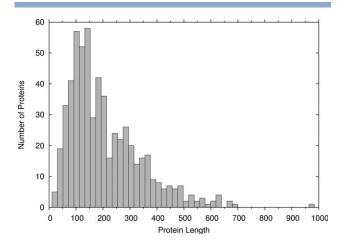
Structural features of proteins were also investigated that can indicate fluctuations represented by B-factors. These features include solvent accessibility of residues, 50 distance from a residue to the center of mass of the protein,⁵¹ eigenvectors of the square distance matrix,⁵² and predicted local fragment structures.⁵³ An alternative direction pursued was to predict B-factors from protein sequences. Machine-learning methods, such as Support Vector Machine, 54,55 the random forest algorithm, 56 or an artificial neural network,⁵⁷ were used to predict fluctuations using sequence information and structural features that can be predicted from sequences, such as the secondary structure and the accessible surface area of residues.

In this work, we used support vector regression (SVR) to investigate the relationship between protein structure and dynamics. We used various structural characteristics as well as structure fluctuation profiles predicted by GNM as input for SVR. The target reference is the dynamics observed in long MD simulations for a representative set of 592 globular proteins. To the best of our knowledge, this is the first time that protein fluctuations have been investigated on such a large dataset of MD simulations. In this context, we also analyzed differences of protein dynamics deducted from the B-factors and the in-solvent dynamics computed by MD simulations. A more practical purpose of this work is to provide a fast (essentially instantaneous in comparison with MD) and reliable method that can be used for predicting fluctuations of protein structures. Unlike existing works mentioned earlier, we predict the real value of residue fluctuations rather than simply showing correlation between predicted and actual fluctuations values. Remarkably, our method predicts fluctuation highly accurately with an average error of less than 1.1 Å. The correlation coefficient of our prediction with the actual fluctuations observed in MD simulations is higher than that of GNM. We also found that some of the static structural features, such as residue contact number, have higher correlation with the residue fluctuation in MD simulation than B-factors do. The developed software for predicting fluctuation, named flexPred, has been made freely available for the academic community.

MATERIALS AND METHODS

Dataset of molecular dynamics trajectories

The molecular dynamics (MD) trajectories of proteins were selected from MoDEL (Molecular Dynamics Extended Library). 58 Of 1897 entries in the database, the following entries were discarded: trajectories for protein structures solved by NMR, those which include more than one protein chain in the simulation, and trajectories for proteins whose length differ from the corresponding entries in the Protein Data Bank (PDB).⁴ These MD



Histogram of the length of proteins in the dataset. There are in total 592 proteins.

trajectories were computed using AMBER,59 GRO-MACS,60 or NAMD61 force fields. If more than one simulation is available for a protein, we used the first one with an earlier entry date in the database. The MoDEL trajectory files were uncompressed with the PCASuite software.⁶² Eight hundred and thirty-seven trajectories remained after this filtering process. From this subset, we removed redundant proteins using the PISCES server⁶³ with a sequence identity cutoff of 35%. The final number of trajectories is 592. This dataset contains proteins from all main classes in the CATH database⁶⁴: 111 proteins in the α class (18.75%), 149 proteins in the β class (25.17%), 256 in the $\alpha\beta$ class (43.24%), and 76 in the few secondary structure class (12.84%). The length of the proteins ranges from 21 to 994 residues (Fig. 1). The simulation time was 10 ns for most of the proteins (96.11%), while the rest of the proteins had shorter trajectories: 5 (0.33%), 2 (2.36%), and 1 ns (0.5%), and one protein each with 6.5, 6.0, 5.5, and 4.5 ns.

Definition of fluctuation

The fluctuation of amino acid residue i is defined in two ways. It can be defined as a root mean square deviation (RMSD) of the mean position of an atom in an MD trajectory:

$$\sqrt{\left\langle \left(\Delta R_i\right)^2 \right\rangle^{MD}} = \sqrt{\frac{1}{T} \sum_{t_j=1}^{T} \left(x_i(t_j) - \left\langle x_i \right\rangle \right)^2}$$
 (1)

where $x_i(t_i)$ is the Cartesian coordinates of the C α atom of residue i at time t_i in the trajectory, T is the number of time frames in the trajectory, and $\langle x_i \rangle$ is the average position of the $C\alpha$ atom of residue i in the trajectory. We also used the coordinates in the PDB file as the refer-

$$\sqrt{\left\langle \left(\Delta R_i\right)^2\right\rangle}^{\text{ref}} = \sqrt{\frac{1}{T} \sum_{t_j=1}^T \left(x_i(t_j) - x_i^{\text{ref}}\right)^2}, \qquad (2)$$

where x_i^{ref} is the coordinates of the $C\alpha$ atom of residue i in the PDB file. The distance of residue positions is computed after superimposing the PDB structure on each frame. If alternative positions of the atom are recorded in the PDB files, the first position of the atom was used. As shown in Figure 2, these two definitions give similar fluctuations of residues, but not identical. The correlation coefficient of the two fluctuation values is 0.86. The fluctuation value is smaller when the mean of a trajectory is used as the reference [Eq. (1)] in almost all the cases (99.9%). Unless noted, we use the second definition of fluctuation [Eq. (2)] in the results that will be shown below, because we compare the fluctuations from MD with B-factors and GNM, both of which are attributed to PDB structures.

Structural features of proteins

We considered the following static protein structural

- 1. B-factor (temperature factor). 28 The B-factor reflects dynamic motion, the static disorder of the atom in the crystal structure, and also errors in model building. The B-factor values are taken from the PDB file.
- 2. Square of the distance between a residue and the protein center of mass, which is defined as follows:

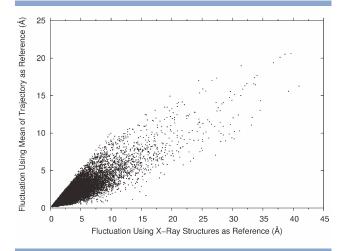


Figure 2 Average fluctuations of proteins in MD trajectories using two definitions. x values show fluctuations of residues relative to the crystal structures of proteins in the PDB [Eq. (2)], while y values are fluctuations relative to the mean structure of each MD trajectory [Eq.

$$r_i^2 = \left(x_i - 1/N \sum_{j=1}^N x_j\right)^2,$$
 (3)

where x_i is the position of the C α atom of residue i. A previous work showed that this parameter has good correlation with the *B*-factor.^{51,52}

- 3. Residue contact number, which is defined as the number of surrounding residues, whose Cα atom is closer than a cutoff distance. The contact number was also shown to correlate well with the *B*-factor. 65,66
- 4. Number of hydrophobic/hydrophilic residue contacts, where the number of residue contacts is separately counted for hydrophobic and hydrophilic residues. Hydrophobic/hydrophilic residues are those which have a positive/negative value on the Kyte-Doolittle hydrophobicity scale.67
- 5. Solvent accessibility surface area (Å²). This parameter is defined as water exposed surface of a residue. We used the DSSP program⁶⁸ to compute the accessibility surface area of amino acids, which are then normalized with the value in the tripeptide with glycines on both sides of the target amino acid residue.⁶⁹
- 6. Residue depth, which is defined as the distance of the $C\alpha$ atom or the average distance of all the atoms in a residue to the closest water molecule. 70 Protein surface was computed with the MSMS program.⁷¹ The hsexpo program was used to compute residue depth.⁷²
- 7. Lower/upper half-sphere exposure of a residue, ⁷² which is defined as the number of contacts within a halfsphere of a radius of 13 Å centering at either the $C\alpha$ or the Cβ atom of the residue. The sphere is divided into half by a plane perpendicular to the $C\alpha$ – $C\beta$ vector.
- 8. Secondary structure. Each residue is classified into eight classes, that is, seven secondary structure types defined by DSSP⁶⁸ or other.
- 9. Fluctuations predicted by the GNM.^{35,36} GNM is a coarse-grained model, where Cα atoms are connected by springs. GNM has been used for investigating protein dynamics including the prediction of B-factor values of proteins.³⁸ We downloaded GNM codes from the Jernigan laboratory (http://ribosome.bb.iastate.edu/). Fluctuations were computed with a residue contact distance cutoff of 16 Å⁷³ and without using cutoff.³⁸ Residue contacts in a protein are represented as the Kirchhoff matrix in GNM:

$$\Gamma_{ij} = \begin{cases} -1 & \text{if} \quad i \neq j \quad \text{and} \quad r_{ij} \leq r_{c} \\ 0 & \text{if} \quad i \neq j \quad \text{and} \quad r_{ij} \geq r_{c} \\ -\sum\limits_{i,i \neq j}^{N} \Gamma_{ij} & \text{if} \quad i = j \end{cases}, \quad (4)$$

where r_{ij} is the distance between two atoms, i and j, and r_c (=16 Å) is the cut-off value. GNM without cutoff uses the following modified Kirchhoff matrix:

$$\Gamma_{ij} = \left\{ \begin{array}{ll} r_{ij}^{-2} & \text{if} \quad i \neq j \\ -\sum\limits_{i,i \neq j}^{N} \Gamma_{ij} & \text{if} \quad i = j \end{array} \right\}.$$
 (5)

In both methods, the average fluctuation of residue i over time is defined by

$$\langle (\Delta R_i)^2 \rangle = C(\Gamma_{ii}^{-1}),$$
 (6)

where C is constant.

Support vector regression

We combined the structural features listed above to predict fluctuations using support vector regression (SVR). The LIBSVM package⁷⁴ with Gaussian kernels was used. Because it was not feasible to test all the possible combinations of features, features were added or changed one at a time starting from the one which has the largest correlation coefficient with residue fluctuation. We performed fivefold cross validation using the dataset of trajectories. The default set of parameters in *libsym*, C = 64.0, $\gamma = 1$, and $\epsilon = 0.5$, was used, which was shown to perform best among others tested in the first few feature combinations in the five-fold cross validation (data not shown).

Evaluation of fluctuations prediction

Pearson's correlation coefficient was used to examine how well individual features or predicted fluctuations correlated with actual fluctuations in the MD trajectories. Average correlation coefficients were computed using all the trajectories in the dataset.

In addition, the error of predicted fluctuations was quantified as the RMSD to the reference trajectory fluc-

RMS =
$$\sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\Delta R_i^{\text{pred}} - \sqrt{\langle (\Delta R_i)^2 \rangle}^{\text{ref}} \right)^2}$$
, (7)

where N is the length of the protein, ΔR_i^{pred} is predicted, and $\sqrt{\left\langle \left(\Delta R_i\right)^2\right\rangle}^{\mathrm{ref}}$ is actual fluctuation [Eq. (2)] of resi-

Availability of the developed program

The program for predicting the fluctuation of residues in a protein structure is made freely available for the academic community at http://kiharalab.org/flexPred/. Both the web server and the source code written in Python are available. It takes a PDB file of a query protein for input data and outputs a predicted fluctuation value for each residue. The computational time for a protein is typically within a couple of seconds to 20 s depending on the length of the protein.

RESULTS AND DISCUSSION

The relationships between structural features and residue fluctuations are examined in several aspects. First, we compare the correlation coefficient of individual static structural features with actual fluctuations. Then, we explore different combinations of features to make accurate prediction of fluctuations using SVR. Then, the accuracy of the fluctuation prediction by SVR and by GNM is further examined. Finally, we also consider the structural variation of models by NMR in comparison with prediction as well as the fluctuations observed in MD trajectories.

Correlation of static structural features of proteins with fluctuations

In Table I, we compared the correlation coefficient of individual structural features with the fluctuation of residues observed in the MD trajectories. Eight different distance cutoff values, 6, 8, to 16 Å, were used for the residue contact number. The top of the table shows the correlation of the B-factor (0.484). Interestingly, several static structural features, namely, the distance to the center of mass and the contact number computed with the cutoff of 12-22 Å, have more significant correlation with the fluctuations than the B-factor. Among the static features, the largest correlation coefficients were observed for the residue contact number (15 and 16 Å). These results indicate that the motion of chains in the MD trajectories is better captured by the coarse-grained topological structures of proteins rather than the *B*-factor.

As a reference, we also show the correlation of the fluctuations predicted by GNM (bottom rows of Table I). GNM showed higher correlation than the other structural features. Note that GNM actually simulates dynamic motion of protein structures; thus, it has a different nature from the other static features compared in the table. Consistently, with the previous work by Yang et al.,³⁸ GNM without using a distance cutoff showed higher correlation than GNM with a distance cutoff.

Because the residue contact number (with a 16 A cutoff) and the square of distance to the center of mass showed two largest correlation coefficients among the static structure features examined, we used these two features as the basis for combinations of input features for training SVR in the next section.

SVR models for predicting residue fluctuation using static structure features

Next, we used SVR to predict the fluctuation of residue positions in the MD trajectories using various combinations of static structural features. Fluctuation predictions by GNM (at the bottom of Table I) were not included as features. Fivefold cross validation was performed, in which SVR parameters were trained on four-

Table I Correlation Coefficients Between Structural Features and Fluctuations

Structural features	Number of proteins with P- <i>value</i> < 0.05 (%) ^a	Avg. corr. coeff. ^b
B-factor	565 (95.4)	0.484 (0.504)
Distance to center of mass	584 (98.6)	0.509 (0.514)
Square of distance to	586 (99.0)	0.545 (0.549)
center of mass		
Contact number (cutoff 6 Å)	571 (96.5)	-0.374 (-0.384)
Contact number (8 Å)	591 (99.8)	-0.480 (-0.481)
Contact number (12 Å)	590 (99.7)	-0.554 (-0.556)
Contact number (15 Å)	587 (99.2)	-0.568 (-0.571)
Contact number (16 Å)	571 (96.5)	-0.567 (-0.571)
Contact number (18 Å)	587 (99.2)	-0.562 (-0.565)
Contact number (20 Å)	585 (98.8)	-0.555 (-0.559)
Contact number (22 Å)	584 (98.6)	-0.545 (-0.551)
Accessible Surface Area (ASA) ^c	580 (98.0)	0.404 (0.407)
ASA normalized	590 (99.7)	0.476 (0.477)
Residue depth (residue mean) ^d	559 (94.4)	-0.352 (-0.371)
Residue depth ($C\alpha$)	553 (93.4)	-0.339 (-0.359)
Half upper sphere exposure $(C\alpha)^e$	568 (95.9)	-0.385 (-0.398)
Half lower sphere exposure (C $lpha$)	567 (95.8)	-0.389 (-0.402)
Half upper sphere exposure (C β)	537 (90.7)	-0.339 (-0.363)
Half lower sphere exposure ($C\beta$)	561 (94.8)	-0.383 (-0.399)
Prediction by GNM (cutoff 16 Å) [†]	586 (99.0)	0.643 (0.648)
Prediction by GNM (no cutoff)	591 (99.8)	0.646 (0.646)

The largest correlation coefficients among the static structural features are highlighted in bold.

^aThe number of proteins that have significant correlation coefficient to the fluctuations (with P-value < 0.05) are counted. The total number of trajectories (proteins) is 592.

The average value calculated only for the subset of proteins with P-value < 0.05 is shown in the parentheses.

^cAccessible surface area (Å²) of amino acid residues without normalization. The next row is the correlation with the normalized accessible surface area.

^dThe residue depth computed as the average distance for each atom in the residue and the distance for the $C\alpha$ atom (next row).

 $^{\mathrm{e}}$ The lower/upper half-sphere exposure of a residue using the $C\alpha$ or the $C\beta$ atom to determine the position of the plane which cut the sphere to half.

^fFluctuations predicted by GNM [Eq. (6)].

fifths of the dataset, while prediction was made for the rest of the one-fifth of the dataset. This procedure was repeated five times to make prediction for all data in the dataset. Starting from the combination of the residue contact number (with 16 Å cutoff) and the square of distance to the center of mass, which are the two features that showed the highest correlation with fluctuations (Table I), 17 different feature combinations were tested by adding one feature at a time (Table II).

Among the 17 feature combinations examined, all except for two (the feature set 1 and set 17) showed higher correlation with actual fluctuations than GNM (Table I). The largest correlation coefficient, 0.669, was achieved for the feature set 15, which uses the residue contact numbers with different distance cutoffs. In terms of average RMS, all the feature combinations predicted residue fluctuations within an RMS of 1.1 Å, ranging from 1.042 to 1.092 Å. The smallest RMS was achieved for feature sets 6, 7, 12, 13, and 14, which combine the residue contact numbers, the square distance from the center of mass, and the B-factor. Sets 6 and 7

Table II Summary of Fluctuation Prediction Using SVR Models with Different Feature Combinations

Feature set	Features used ^a	Number of proteins with P-value < 0.05 (%)	Average corr. coeff.b	RMS (Å) ^c
1	C(16), D ²	584 (98.6)	0.638 (0.644)	1.075
2	C(16), D ² , B	587 (99.2)	0.654 (0.658)	1.067
3	C(16), D ² , B, C(18)	587 (99.2)	0.655 (0.659)	1.060
4	C(16), D ² , B, C(18), Sec	589 (99.5)	0.661 (0.664)	1.048
5	C(16), D ² , B, C(18), Res-type	586 (99.0)	0.652 (0.657)	1.063
6	C(16), D ² , B, C(18), Sec, C(12)	589 (99.5)	0.665 (0.668)	1.042
7	C(16), D ² , B, C(18), Sec, C(12), C(8)	588 (99.3)	0.667 (0.668)	1.042
8	C(16), D ² , C(18), C(12), C(8), C(6)	588 (99.3)	0.656 (0.660)	1.053
9	C(16), D ² , B, C(18), C(12), C(8), C(6)	588 (99.3)	0.666 (0.669)	1.045
10	C(16), D ² , B, C(18), C(12), C(8), C(6), Sec	589 (99.5)	0.665 (0.667)	1.043
11	C(16), D ² , B, C(18), C(12), C(8), C(6), Acc	587 (99.2)	0.665 (0.669)	1.045
12	C(16), D ² , B, C(18), C(12), C(8), C(6), C(20)	588 (99.3)	0.666 (0.670)	1.042
13	C(16), D ² , B, C(18), C(12), C(8), C(6), C(20), C(22)	588 (99.3)	0.667 (0.670)	1.042
14	C(16), D ² , B, C(18), C(12), C(8), C(6), C(15), C(20), C(22)	588 (99.3)	0.666 (0.670)	1.042
15	C(16), B, C(18), C(12), C(8), C(6), C(20), C(22)	588 (99.3)	0.669 (0.673)	1.073
16	C(16), C(18), C(12), C(8), C(6), C(15), C(20), C(22)	587 (99.2)	0.660 (0.665)	1.092
17	C(16), B, C(18), C(12), C(8), C(6), C(20), C(22), HP	587 (99.2)	0.647 (0.651)	1.092

The largest correlation coefficients among the static structural features are highlighted in bold.

additionally used information about the secondary structure. The RMS and the average correlation coefficients (Table II) correlate moderately with a correlation coefficient of 0.627 (Fig. 3). Figure 4 shows the distribution of the average correlation coefficients between predicted and actual fluctuations [Fig. 4(A)] and the average RMS [Fig. 4(B)] for each protein, which were predicted using feature set 12. Remarkably, the majority (70%) of proteins fluctuations were predicted within an RMS of 1.0 Å. The strong advantage of the developed SVR models is that

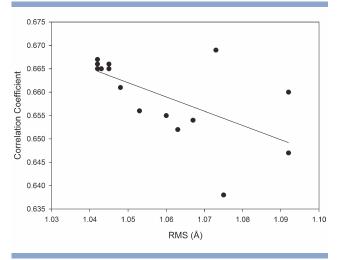
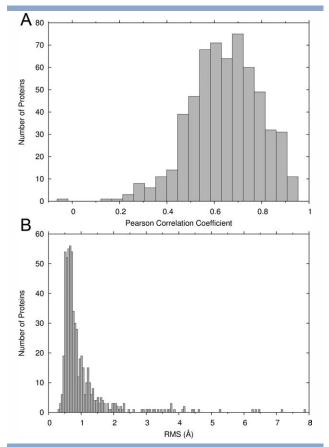


Figure 3 The average correlation coefficient and RMS of predicted and actual fluctuations. Predictions were made with SVR using 17 different feature combinations (Table II).



Distribution of (A), corelation coefficients; (B), RMS (Å) of predicted and actual fluctuations computed for 592 proteins in the dataset.

aC(x), the residue contact number with x Å distance cutoff; B, B-factor; D², square of the distance between the Cα atom to the protein center of mass; Sec, the secondary structure; Acc, normalized accessible surface area; HP, the number of hydrophilic/hydrophobic contacts, Res-Type, amino acid type of residues.

^bThe average correlation coefficients between predicted and actual fluctuations. Values calculated only for the subset of proteins that have significant correlation with P-value < 0.05 is shown in the parentheses.

^cThe RMS [Eq. (7)] was averaged over all the proteins in the dataset.

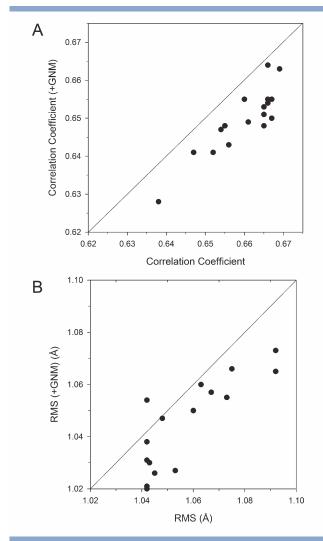


Figure 5Comparison of the prediction performance with and without using GNM as a feature. $\langle (\Delta R_i)^2 \rangle$ predicted by GNM was added to each SVR feature set listed in Table II. (A) Average correlation coefficient; (B) average RMS predicted by SVR with and without $\langle (\Delta R_i)^2 \rangle$ from GNM are plotted.

they predict the real value of fluctuation, unlike GNM, which predicts only the relative magnitude of residue

fluctuations that need to be rescaled to obtain actual fluctuation values.

Incorporating dynamic features to SVR models

We further investigated whether adding GNM as an input feature can improve fluctuations prediction with SVR. We used $\langle (\Delta R_i)^2 \rangle$ for the fluctuations from GNM [Eq. (6)] without a distance cutoff, because it has higher correlation with the actual fluctuations than $\sqrt{\langle (\Delta R_i)^2 \rangle}$ does. To each of the feature sets examined in Table II, we added $\langle (\Delta R_i)^2 \rangle$ predicted by GNM and performed fivefold cross validation. The resulting fluctuation prediction with and without GNM was compared in terms of the correlation coefficient [Fig. 5(A)] and the RMS [Fig. 5(B)] with the actual fluctuations.

Adding GNM in the feature set made slight improvement in the RMS of the predicted fluctuations [Fig. 5(B)] except for one case (feature set 12), lowering RMS on average by 0.010. However, small consistent deterioration of the correlation coefficient was observed [Fig. 5(A)] when GNM was added. The average decrease in the correlation coefficient is 0.013. Thus, GNM did not make significant contribution to improving fluctuation prediction.

Comparison of SVR model prediction results with *B*-factor fluctuation values

In Figure 6, we show four examples of actual and predicted fluctuations as well as fluctuations derived from the *B*-factors. For residue i with a *B*-factor of B_{i} , the fluctuation is defined as

$$\sqrt{\left\langle \left(\Delta R_{i}\right)^{2}\right\rangle }^{\mathrm{Bfactor}}=\sqrt{\frac{3\mathrm{B}_{i}}{8\pi^{2}}}.$$
 (8)

The fluctuations from the *B*-factor were also rescaled to achieve a smaller RMS with the actual fluctuations (i.e., fluctuations from MD trajectories) as follows

$$\sqrt{\left\langle \left(\Delta R_{i}\right)^{2}\right\rangle _{\text{rescaled}}} = \sqrt{\left\langle \left(\Delta R\right)^{2}\right\rangle _{\text{min}}} + \alpha \left(\sqrt{\left\langle \left(\Delta R\right)^{2}\right\rangle _{\text{max}}} - \sqrt{\left\langle \left(\Delta R\right)^{2}\right\rangle _{\text{min}}}\right) \frac{\sqrt{\left\langle \left(\Delta R_{i}\right)^{2}\right\rangle _{\text{Bfactor}}^{\text{Bfactor}}} - \sqrt{\left\langle \left(\Delta R\right)^{2}\right\rangle _{\text{min}}^{\text{Bfactor}}}}{\sqrt{\left\langle \left(\Delta R\right)^{2}\right\rangle _{\text{max}}^{\text{Bfactor}}} - \sqrt{\left\langle \left(\Delta R\right)^{2}\right\rangle _{\text{min}}^{\text{Bfactor}}}},$$
(9)

where $\sqrt{\left\langle (\Delta R)^2 \right\rangle}_{max}$ and $\sqrt{\left\langle (\Delta R)^2 \right\rangle}_{min}$ are the maximum and the minimum values of actual fluctuations, and $\sqrt{\left\langle (\Delta R)^2 \right\rangle}_{max}^{Bfactor}$ and $\sqrt{\left\langle (\Delta R)^2 \right\rangle}_{min}^{Bfactor}$ are the maximum and the minimum fluctuation values computed from *B*-factor values [Eq. (8)] in the protein. α is a weighting factor explored from 0.1 to 1.0 with an interval of 0.1 to seek

smaller RMS for the actual fluctuations (Table III). In Figure 6, α is set to 1.0 for the plots of "Fluctuation from *B*-factor, rescaled." Note that this rescaling obviously changes the RMS but does not change the correlation coefficient to the actual fluctuation. The acutal fluctuations in the MD trajectories are defined by Eq. (2), and predictions were made using feature set 15 in Table II. The right panel of

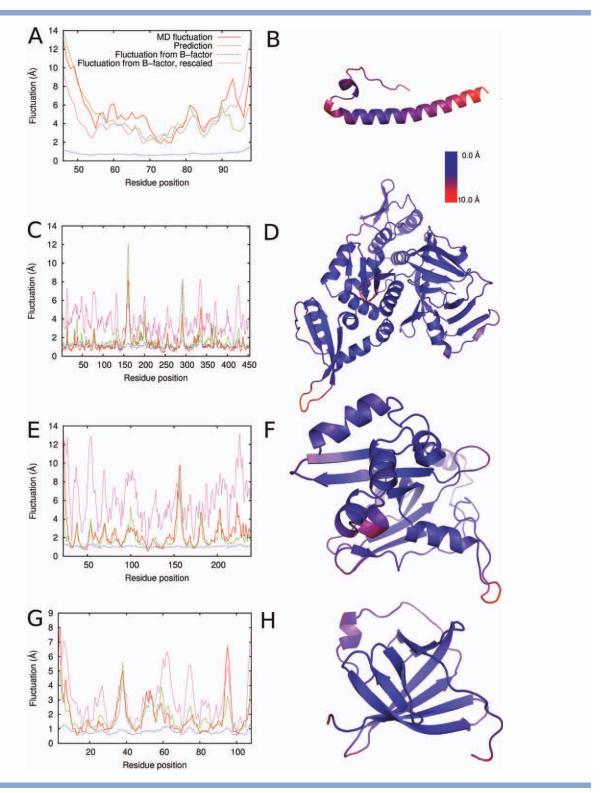


Figure 6 Examples of predicted fluctuations in comparison with B-factor-derived fluctuations and MD simulation fluctuations. Left panels show the values of fluctuations: red, fluctuations observed in the MD trajectories; green, predicted fluctuations; dotted blue line, fluctuations computed from *B*-factors; dotted magenta line, rescaled fluctuations from *B*-factors ($\alpha = 1.0$). The correlation coefficients and RMS are summarized in Table III. Right-hand panels show the magnitude of fluctuations in a color scale with blue indicating lower fluctuations and red for higher fluctuations. **A**, **B**, 1mof; C, D, 1dq3; E, F, 1gpc; G, H, 1a1x.

Table III Correlation Coefficients and RMS of the Four Example Predictions

	Correlatio	Correlation coefficient		RMS (Å)		
PDB ID	B-factor	Prediction	B-factor	B-factor, rescaled $\alpha = 1.0^a$	B-factor, rescaled $(\alpha)^b$	Prediction
1mof	0.69	0.80	4.92	1.91	1.91 (1.0)	1.55
1dq3	0.50	0.81	0.94	2.64	0.85 (0.4)	0.71
1gpc	0.55	0.78	1.93	4.32	1.42 (0.4)	1.04
1alx	0.61	0.82	1.60	1.72	1.09 (0.6)	0.79

The data correspond to plots at the left panels in Figure 6.

 a Fluctuations computed from B-factor were rescaled with $\alpha=1.0$ in [Eq. (9)]. This value corresponds to the curve "Fluctuation from B-factor, rescaled" in Figure 6. bFluctuations computed from B-factor were rescaled with the weight factor α [Eq. (9)] ranging from 0.1 to 1.0 with an interval of 0.1. Then the smallest RMS obtained is shown together with the used α value in the parentheses.

each protein visualizes the magnitude of actual fluctuations in a color scale from blue to red with blue showing small while red for large fluctuation.

The first example, retrovirus coat protein (PDB ID: 1mof) [Fig. 6(A,B)], exhibits a large fluctuation at two termini and at the end of the long helix. Prediction by SVR captured fluctuating residues and the magnitude fairly well with a correlation coefficient of 0.80 and an RMS of 1.55 Å. The fluctuations derived from B-factor have lower correlation with the actual fluctuations (correlation coefficient of 0.69) with a larger RMS of 1.91 Å even after rescaling. In the second example [Fig. 6(C,D)] of homing endonuclease PI-PfuI (PDB ID: 1dq3), overall fluctuation is not large but shows high peaks of fluctuation at loop regions. The predicted fluctuations have a correlation coefficient of 0.81 while the fluctuations from B-factor have a moderate correlation of 0.50. The third example, DNA-binding protein gp32 (PDB ID: 1gpc) [Fig. 6(E,F)], has the largest fluctuation at the loop of residues 150-160 and over 3 Å fluctuation at the other loop regions, which are captured well by the prediction. Predicted fluctuations have a correlation coefficient of 0.78 and a small RMS of 1.04 Å. In contrast, the correlation of fluctuations from B-factor is 0.55 with a larger RMS of 1.93 Å. The last example, MTCP-1 (PDB ID: 1a1x) [Fig. 6(G,H)], is a β -barrel protein with a long loop at residues 50-60. Relatively large fluctuation was observed at the N-terminus and at the loop regions that connect β-strands (e.g., residues 35-40), which are well predicted. The overall RMS of the prediction is 0.79 Å, and the correlation coefficient with the actual fluctua-

Table IV Comparison of Fluctuations of NMR Models, MD, and Our Prediction

	Number of proteins with		,
Compared data	P-value < 0.05 (%)	Corr. coeff.	RMS (Å)
NMR versus MD	136 (97.1)	0.651 (0.667)	2.425
NMR versus prediction	138 (98.6)	0.739 (0.747)	1.808
MD versus prediction	138 (98.6)	0.686 (0.693)	2.165

Hundred and forty nonredundant proteins in the MoDEL database were used whose structures were determined by NMR.

tions is 0.82, better than the fluctuations derived from Bfactors

Consistent with Table I, the fluctuations from B-factors correlate only moderately with the actual fluctuations. Fluctuations computed from B-factors using Eq. (8) have always a larger RMS than the SVR prediction. The agreement of the fluctuations from B-factors can be improved if it is rescaled individually for each protein as shown in the second column from the right in Table III; however, the value of the optimal scaling factor α differs from protein to protein and thus cannot be known beforehand. In contrast, our prediction by SVR has a significantly higher correlation with the actual prediction, and it predicts the real value of the fluctuations satisfactorily without any rescaling.

MD fluctuations and fluctuations from NMR models

The MoDEL database also contains simulations of protein structures determined by NMR. We selected 140 nonredundant protein structures determined by NMR that contain more than 10 models in their PDB files. Redundant proteins were removed by considering sequence identity according to the PISCES database.⁶³ Using the 140 proteins, we compared fluctuations observed in the NMR models, MD trajectories, and the predicted fluctuations. The results are summarized in Table IV. The fluctuation prediction was carried out using feature set 16, which does not contain the B-factor term (NMR structures do not have B-factors).

It is shown that the prediction has a significant correlation (0.739) with the structural variation of the models derived from NMR. Interestingly, the correlation coefficient between the prediction and NMR is highest among the other two pairs, prediction versus MD and NMR versus MD.

CONCLUSION

We used a large number of MD trajectories of nonhomologous proteins as references and examined static structural features of the proteins that are most relevant

to fluctuations. We examined the correlation of individual structural features with fluctuations and then investigated effective combinations of features for SVR to predict the real value of fluctuation of residues. The main findings of this work are summarized as follows. First of all, two types of structural features, the distance to the center of mass of the protein and the residue contact number, showed a higher correlation coefficient with fluctuations than B-factor does. Combinations of static features used as input for SVR achieved accurate prediction of fluctuations with a correlation coefficient of 0.67 and RMS of 1.042 Å. This correlation coefficient is higher than GNM to the actual fluctuation. Our method predicts the structural variation of NMR models also well. The current study demonstrates that flexibility of proteins is inherently coded in coarse-grained static protein structural features, even more than in the crystallographic B-factors. Thus, protein motion is determined by its static structure that is coded by its sequence, which could be considered as an extension of the Anfinsen's dogma.⁷⁵ Indeed, series of studies on GNM has also demonstrated that motion of a protein is determined by its structure. However, the current work further shows that static structural features can predict the real value of fluctuations, which GNM has not been shown to be able to do. As the importance of protein dynamics has been more recognized for biological function, the prediction method we developed has also a practical value in the wide areas of biology and biotechnology.

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